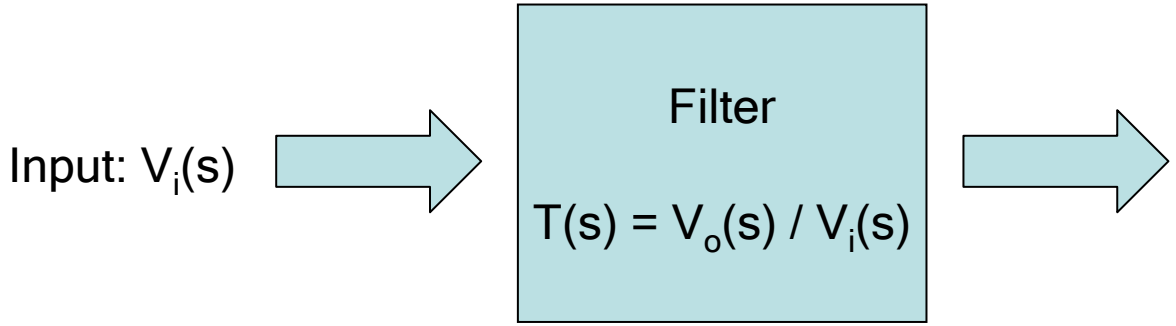
Lect. 14: First-Order Filters

Filter: A device that alters the frequency spectrum of signals passing through it

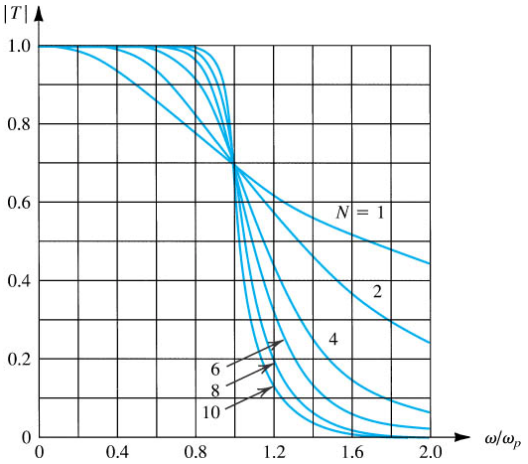


$$T(s) = \frac{a_M S^M + a_{M-1} S^{M-1} + \cdots + a_0}{S^N + b_{N-1} S^{N-1} + \cdots + b_0} = \frac{a_M (s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)}$$

N: filter order

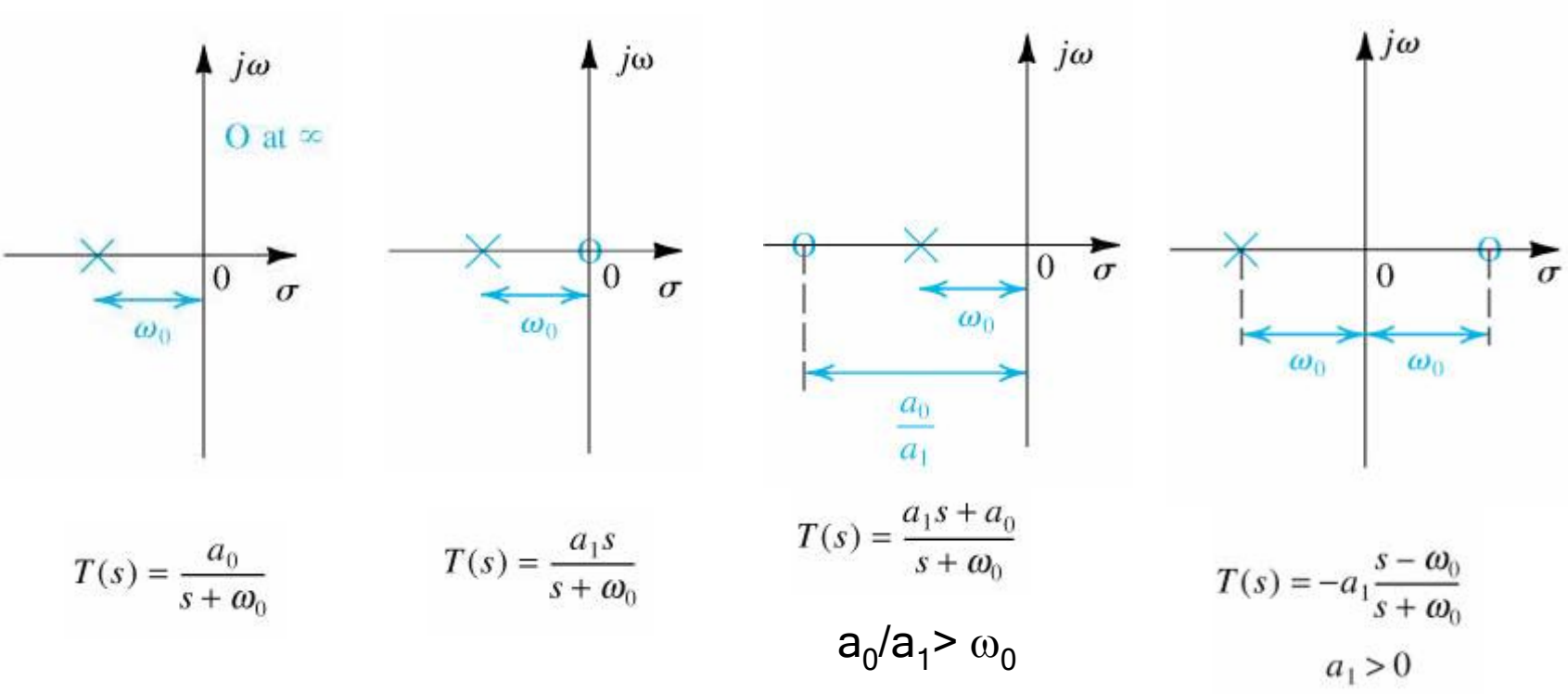
poles, zeros

$$M \leq N$$



Lect. 14: First-Order Filters

First-order filter $T(s) = \dfrac{a_1 s + a_0}{s + \omega_0}$



$T(s) = \dfrac{a_0}{s + \omega_0}$

$T(s) = \dfrac{a_1 s}{s + \omega_0}$

$T(s) = \dfrac{a_1 s + a_0}{s + \omega_0}$

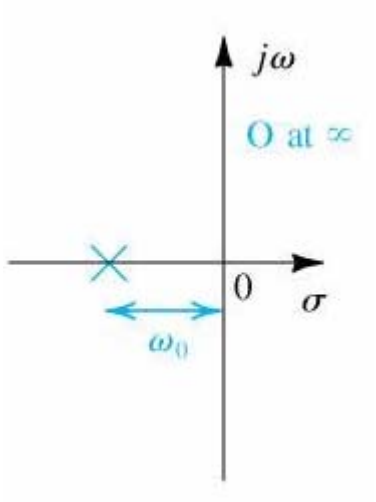
$a_0/a_1 > \omega_0$

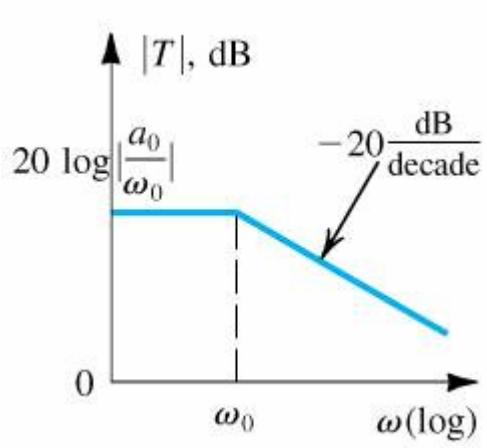
$T(s) = -a_1 \dfrac{s - \omega_0}{s + \omega_0}$

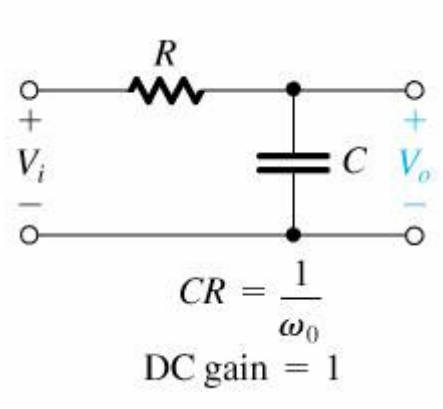
$a_1 > 0$

Filter characteristics depend on pole, zero locations

Lect. 14: First-Order Filters



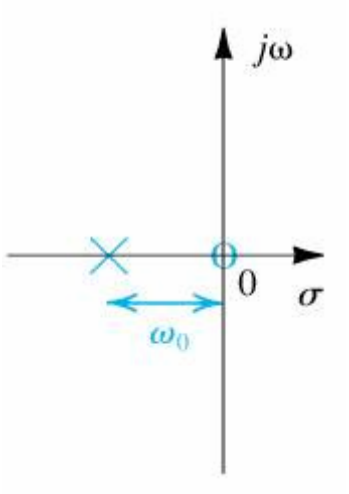


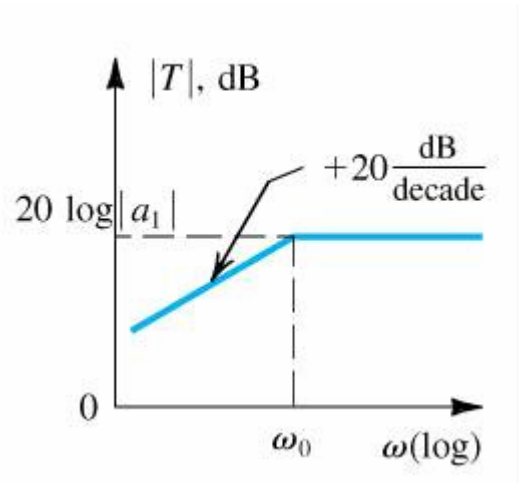


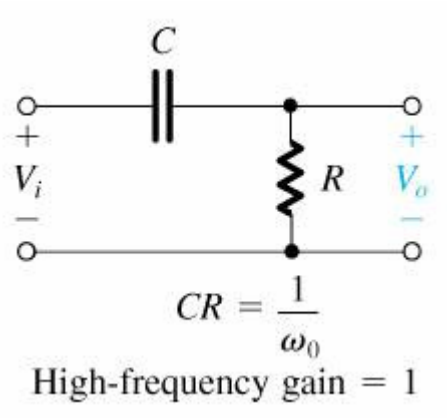
$$T(s) = \frac{a_0}{s + \omega_0}$$

Low Pass (LP) Filter

Lect. 14: First-Order Filters



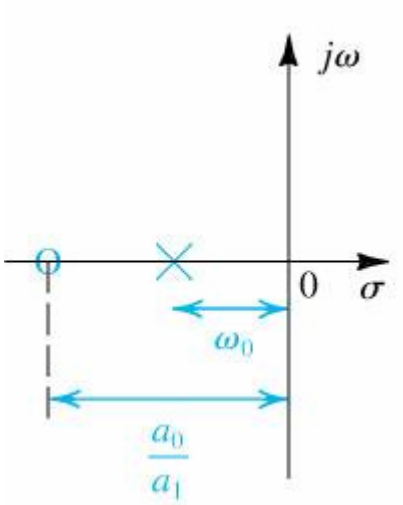




$$T(s) = \frac{a_1 s}{s + \omega_0}$$

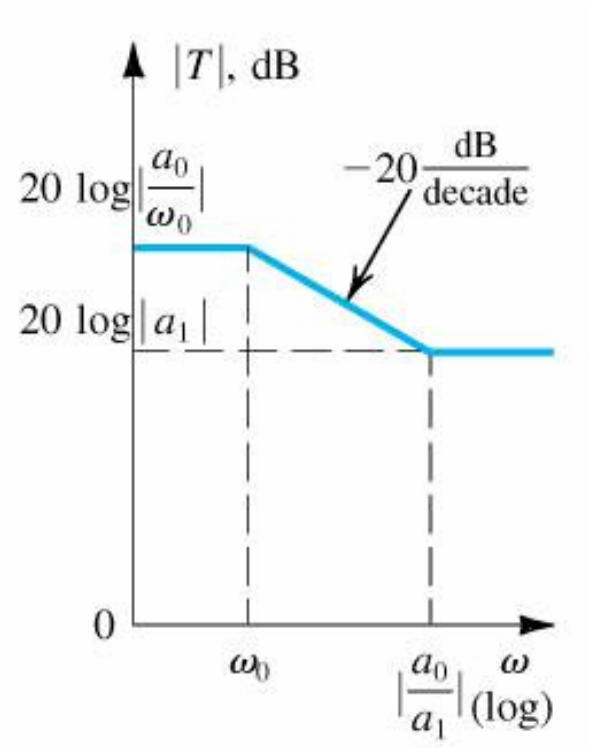
High Pass (HP) Filter

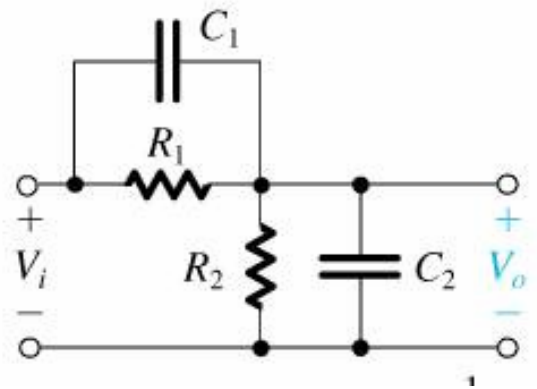
Lect. 14: First-Order Filters



$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

$a_0/a_1 > \omega_0$





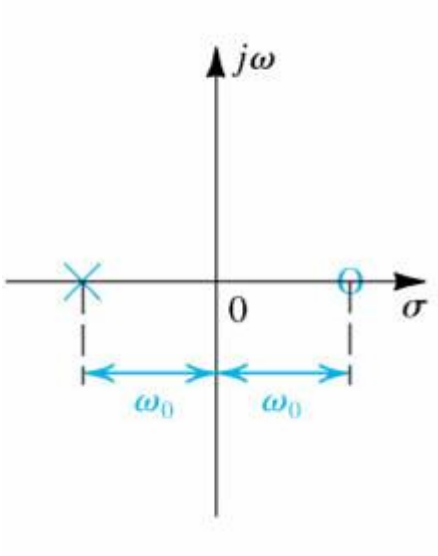
$$(C_1 + C_2)(R_1 /\!/ R_2) = \frac{1}{\omega_0}$$

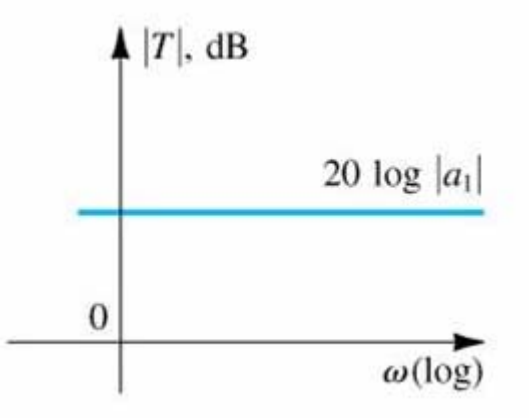
$$C_1 R_1 = \frac{a_1}{a_0}$$

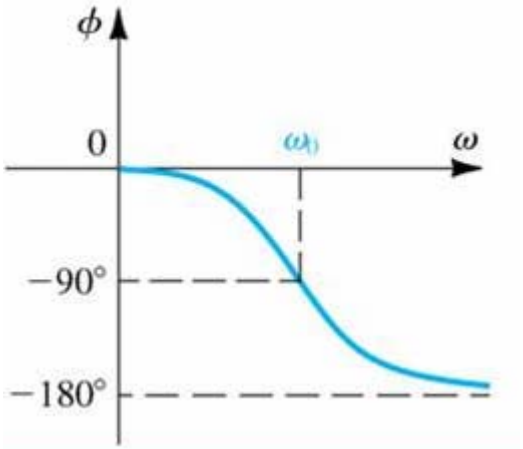
$$\text{DC gain} = \frac{R_2}{R_1 + R_2}$$

$$\text{HF gain} = \frac{C_1}{C_1 + C_2}$$

Lect. 14: First-Order Filters





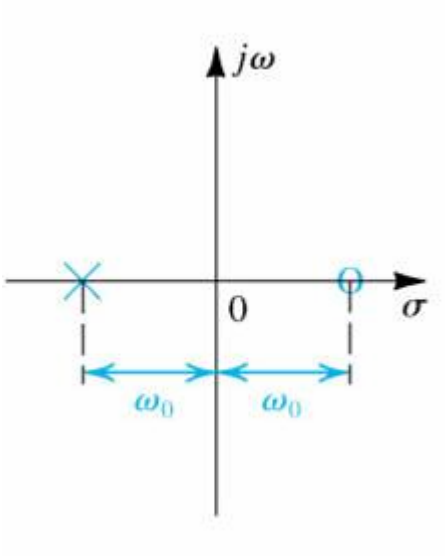


$$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$$

$$a_1 > 0$$

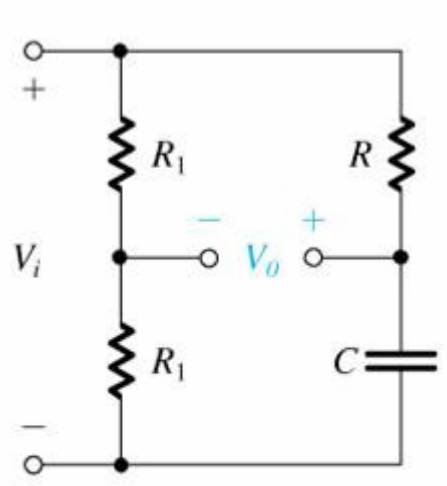
All Pass (AP) Filter:
Phase change only ➔ Delay in time domain

Delay line



$$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$$

$$a_1 > 0$$



$CR = 1/\omega_0$

Flat gain $(a_1) = 0.5$

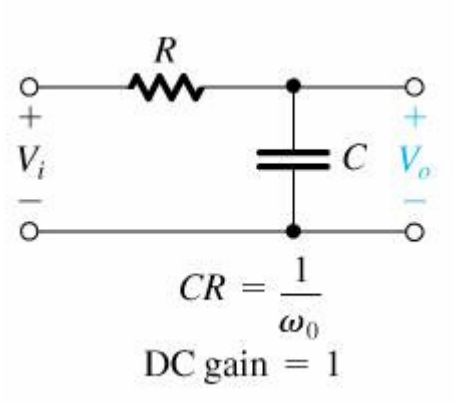
$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} - \frac{1}{2}$$

$$= \frac{1}{1 + sRC} - \frac{1}{2} = \frac{1 - sRC}{2 + 2sRC}$$

$$= -\frac{1}{2} \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$

Lect. 14: First-Order Filters

Limitation of passive filters (RC): Fixed gain, Loading effect



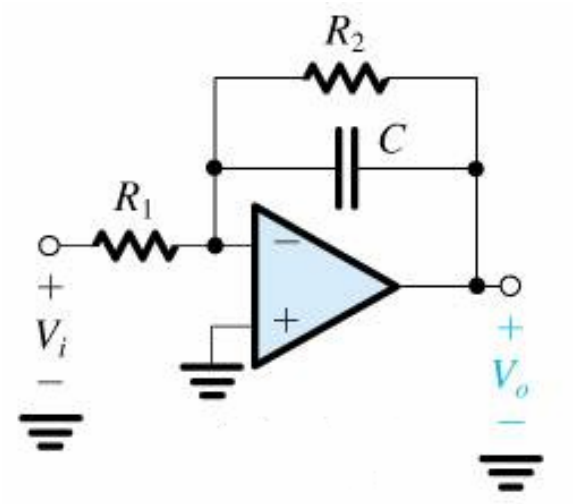
$$CR = \frac{1}{\omega_0}$$

DC gain = 1

$$T(s) = \frac{a_0}{s + \omega_0}$$

LP Filter

➔ Active filters



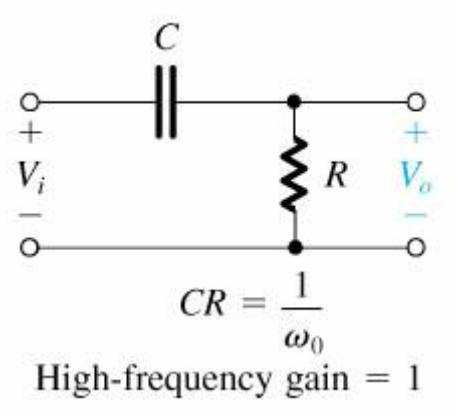
$$V_o(s) = -\frac{V_i(s)}{R_1} \cdot R_2 \,//\, \frac{1}{sC}$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{R_2}{R_1}\frac{1}{sR_2C + 1}$$

$$CR_2 = \frac{1}{\omega_0}$$

$$\text{DC gain} = -\frac{R_2}{R_1}$$

Lect. 14: First-Order Filters

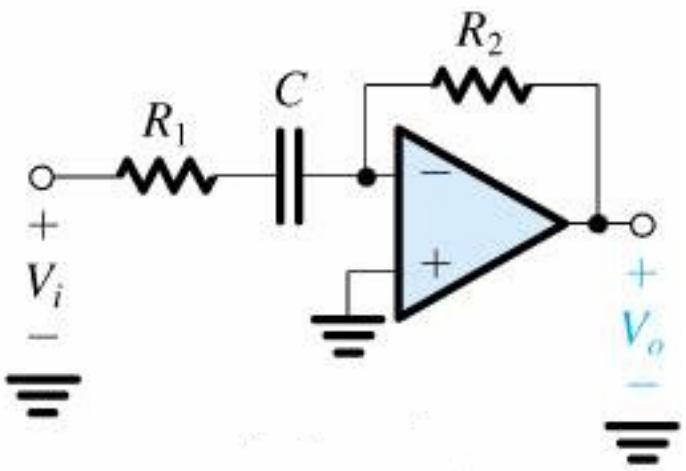


$$CR = \frac{1}{\omega_0}$$

High-frequency gain = 1

$$T(s) = \frac{a_1 s}{s + \omega_0}$$

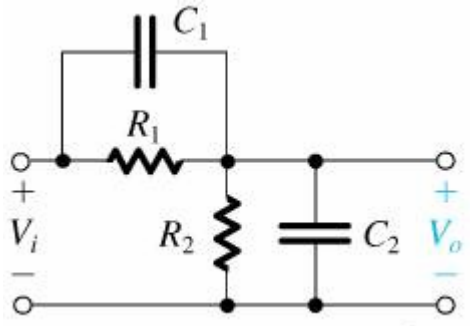
High Pass (HP) Filter



$$CR_1 = \frac{1}{\omega_0}$$

High-frequency gain $= -\frac{R_2}{R_1}$

Lect. 14: First-Order Filters

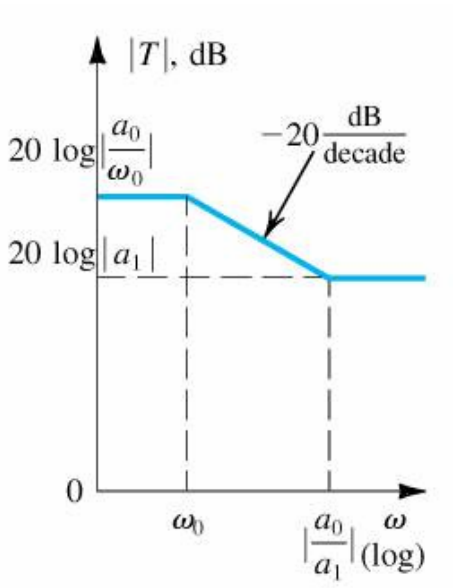


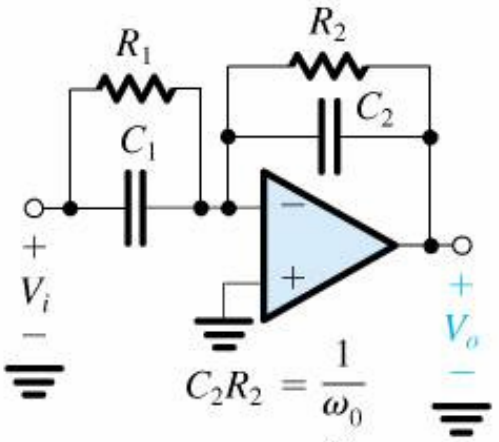
$$(C_1 + C_2)(R_1 /\!/ R_2) = \frac{1}{\omega_0}$$

$$C_1R_1 = \frac{a_1}{a_0}$$

$$\text{DC gain} = \frac{R_2}{R_1 + R_2}$$

$$\text{HF gain} = \frac{C_1}{C_1 + C_2}$$





$$C_2R_2 = \frac{1}{\omega_0}$$

$$C_1R_1 = \frac{a_1}{a_0}$$

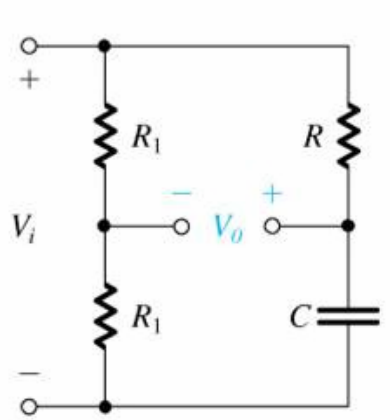
$$\text{DC gain} = -\frac{R_2}{R_1}$$

$$\text{HF gain} = -\frac{C_1}{C_2}$$

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

$a_0/a_1 > \omega_0$

Lect. 14: First-Order Filters

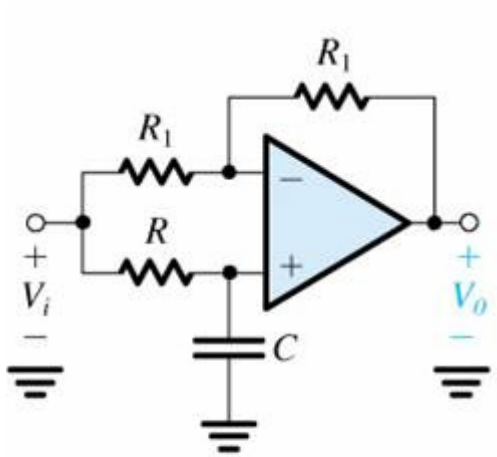


$CR = 1/\omega_0$

Flat gain $(a_1) = 0.5$

$$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$$

$$a_1 > 0$$



$CR = 1/\omega_0$

Flat gain $(a_1) = 1$

$$V^+ = V_i \frac{1/sC}{1/sC + R}$$

$$V^- = \frac{V_i + V_o}{2}$$

$$V_i \frac{2}{1 + sRC} = V_i + V_o$$

$$V_o = V_i\left(\frac{2}{1 + sRC} - 1\right)$$

$$\frac{V_o}{V_i} = \frac{2 - 1 - sRC}{1 + sRC} = -\frac{sRC - 1}{sRC + 1}$$